

## 6. Feladat

## (Kétváltozós hiperbolikus programozási feladat grafikus vizsgálata)

Grafikusan vizsgáljuk meg az  $(E, z)$  maximumfeladatot és az  $(E, z)$  minimumfeladatot! Egynél több megoldás létezése esetén nevezzük meg a végtelen megoldáshalmaz egész-komponensű vektorait a hozzájuk tartozó eltérésvektorokkal együtt (vagyis adjuk meg a feladat összes egészértékű megoldását)!

$$1. \quad E: \begin{cases} 3x_1 - 2x_2 \leq 12 \\ -x_1 + x_2 \leq 6 \\ x_1 + 3x_2 \geq 6 \\ x_2 \leq 9 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{6x_1 + x_2 - 4}{11x_1 + x_2 + 1}$$

$$2. \quad E: \begin{cases} x_1 + x_2 \geq 4 \\ -3x_1 + x_2 \leq 0 \\ x_1 \leq 9 \\ x_1 + 2x_2 \geq 6 \\ x_2 \leq 7 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 + 3}{x_1 + x_2 + 1}$$


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$$3. \quad E: \begin{cases} -2x_1 + x_2 \leq 2 \\ 3x_1 + 2x_2 \geq 6 \\ 2x_1 - x_2 \leq 8 \\ x_1 + 5x_2 \geq 5 \\ x_1 \leq 6 \\ x_2 \leq 6 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + x_2 + 1}{5x_1 + 7x_2 + 1}$$

$$4. \quad E: \begin{cases} x_1 - 2x_2 \leq 4 \\ 3x_1 + 2x_2 \geq 6 \\ 5x_1 + 8x_2 \leq 40 \\ -x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-8x_1 + x_2 - 3}{x_1 + x_2 + 6}$$


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$$5. \quad E: \begin{cases} 5x_1 - 3x_2 \leq 15 \\ x_1 + x_2 \geq 2 \\ 5x_1 + 6x_2 \leq 60 \\ x_1 \leq 10 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 3x_2 + 1}{5x_1 + 7x_2 + 1}$$

$$6. \quad E: \begin{cases} -x_1 + x_2 \leq 1 \\ -x_1 + 2x_2 \geq 0 \\ x_1 \leq 2 \\ 3x_1 - x_2 \geq 0 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{4x_1 + 3x_2 + 1}{x_1 + x_2 + 1}$$


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$$7. \quad E: \begin{cases} x_1 + 4x_2 \geq 8 \\ -3x_1 + x_2 \leq 6 \\ -x_1 + 2x_2 \geq -4 \\ 5x_1 + x_2 \geq 5 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-3x_1 + 2x_2 - 9}{4x_1 + x_2 + 1}$$

$$8. \quad E: \begin{cases} x_1 + 10x_2 \geq 10 \\ 4x_1 + x_2 \geq 8 \\ x_1 - x_2 \geq -3 \\ x_1 + x_2 \leq 7 \\ 3x_1 - 4x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-3x_1 + x_2 + 13}{x_1 + x_2 + 1}$$


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$$9. \quad E: \begin{cases} -x_1 + 2x_2 \leq 4 \\ 2x_1 - x_2 \leq 8 \\ x_1 + x_2 \geq 1 \\ -x_1 + x_2 \geq -3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + x_2 + 2}{3x_1 + 2x_2 + 1}$$

$$10. \quad E: \begin{cases} x_1 - 2x_2 \geq -4 \\ x_1 + x_2 \geq 1 \\ x_2 \leq 3 \\ x_1 - x_2 \leq 3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + x_2 - 1}{9x_1 + 2x_2 + 4}$$

$$11. \quad E: \begin{cases} 5x_1 + 6x_2 \leq 60 \\ x_1 + x_2 \geq 3 \\ 5x_1 - 3x_2 \leq 15 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 3x_2 + 1}{5x_1 + 7x_2 + 1} \rightarrow \max.$$

$$12. \quad E: \begin{cases} x_1 - 2x_2 \leq 8 \\ -x_1 + x_2 \leq 4 \\ x_1 \geq 3 \\ x_1 + x_2 \geq 6 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-4x_1 + x_2 + 11}{4x_1 + 3x_2 + 1}$$

$$13. \quad E: \begin{cases} x_1 + x_2 \geq 4 \\ 3x_1 - x_2 \geq 0 \\ x_1 + 2x_2 \geq 6 \\ x_1 - x_2 \leq 3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-4x_1 + x_2 + 11}{4x_1 + 3x_2 + 1}$$

$$14. \quad E: \begin{cases} x_1 + x_2 \geq 2 \\ -2x_1 + x_2 \leq 2 \\ x_1 \leq 5 \\ x_1 + 4x_2 \geq 4 \\ x_2 \geq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + x_2 + 3}{3x_1 + 2x_2 + 1}$$

$$15. \quad E: \begin{cases} 3x_1 - 2x_2 \leq 18 \\ x_2 \leq 9 \\ -x_1 + x_2 \leq 6 \\ x_1 + 3x_2 \geq 6 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 11x_2 + 3}{x_1 + 10x_2 + 1}$$

$$16. \quad E: \begin{cases} -5x_1 + 2x_2 \leq 10 \\ 3x_1 - 7x_2 \leq 21 \\ 3x_1 + 4x_2 \geq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-3x_1 + 2x_2 - 24}{2x_1 + x_2 + 2}$$

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$$17. \quad E: \begin{cases} 5x_1 + 6x_2 \leq 60 \\ x_1 + x_2 \geq 3 \\ 5x_1 - 3x_2 \leq 15 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 5x_2 - 1}{x_1 + 2x_2 + 1}.$$

$$18. \quad E: \begin{cases} x_1 + 2x_2 \geq 4 \\ -2x_1 + x_2 \leq 4 \\ 3x_1 - 4x_2 \leq 12 \\ x_1 \leq 8 \\ x_2 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{5x_1 + x_2 + 7}{3x_1 + x_2 + 1}.$$

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$$19. \quad E: \begin{cases} x_1 + x_2 \geq 6 \\ x_1 - 2x_2 \leq 8 \\ -x_1 + x_2 \leq 4 \\ x_1 \geq 3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 - 2x_2 + 11}{2x_1 + 5x_2 + 1}.$$

$$20. \quad E: \begin{cases} 2x_1 - x_2 \leq 2 \\ 3x_1 + 2x_2 \leq 18 \\ x_1 + x_2 \geq 1 \\ x_2 \leq 6 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 4x_2 - 3}{x_1 + x_2 + 1}.$$

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$$21. \quad E: \begin{cases} 4x_1 + 3x_2 \leq 24 \\ x_1 + x_2 \geq 3 \\ 4x_1 - 3x_2 \leq 3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 - x_2 - 7}{2x_1 + x_2 + 1}.$$

$$22. \quad E: \begin{cases} 2x_1 + x_2 \leq 8 \\ x_1 + x_2 \leq 6 \\ -x_1 + x_2 \leq 5 \\ x_1 + 2x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 + 4}{5x_1 + 9x_2 + 1}.$$

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$$23. \quad E: \begin{cases} x_1 - x_2 \leq 3 \\ -x_1 + 3x_2 \leq 3 \\ 3x_1 + 2x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 + 10}{x_1 + x_2 + 4}.$$

$$24. \quad E: \begin{cases} x_1 - x_2 \leq 3 \\ x_1 + x_2 \geq 5 \\ -2x_1 + x_2 \leq 6 \\ x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + 3x_2 + 10}{2x_1 + x_2 + 10}.$$


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$$25. \quad E: \begin{cases} 3x_1 + 2x_2 \geq 12 \\ -4x_1 + x_2 \leq 4 \\ x_1 + 2x_2 \geq 8 \\ x_1 - 3x_2 \leq 3 \\ x_1 \leq 12 \\ x_2 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-x_1 - 2x_2 - 7}{x_1 + x_2 + 6}.$$

$$26. \quad E: \begin{cases} 7x_1 + 6x_2 \leq 42 \\ -5x_1 + 3x_2 \leq 15 \\ 3x_1 - 4x_2 \leq 12 \\ 2x_1 + x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{9x_1 + 4x_2 - 1}{5x_1 + 2x_2 + 1}.$$

$$27. \quad E: \begin{cases} -x_1 + x_2 \geq -3 \\ -2x_1 + x_2 \leq 6 \\ x_1 + x_2 \geq 5 \\ x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{7x_1 - 3x_2 + 13}{5x_1 + 2x_2 + 1}.$$

$$28. \quad E: \begin{cases} 2x_1 + x_2 \leq 8 \\ x_1 + x_2 \leq 6 \\ -x_1 + x_2 \leq 5 \\ x_1 + 2x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 + 4}{5x_1 + 9x_2 + 1}.$$

$$29. \quad E: \begin{cases} -3x_1 + 5x_2 \leq 15 \\ x_1 + x_2 \geq 2 \\ 6x_1 + 5x_2 \leq 60 \\ x_2 \leq 5 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-2x_1 - 3x_2 + 1}{5x_1 + 9x_2 + 2}.$$

$$30. \quad E: \begin{cases} x_1 - 2x_2 \leq 8 \\ x_1 + x_2 \geq 6 \\ x_1 \geq 3 \\ -x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 4x_2 + 1}{5x_1 + 7x_2 + 1}.$$

$$31. \quad E: \begin{cases} -3x_1 + 5x_2 \leq 15 \\ 6x_1 + 5x_2 \leq 60 \\ x_1 + x_2 \geq 2 \\ x_2 \leq 10 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{7x_1 + 5x_2 + 1}{3x_1 + 2x_2 + 1}.$$

$$32. \quad E: \begin{cases} x_1 - 2x_2 \leq 8 \\ x_1 + x_2 \geq 6 \\ x_1 \geq 3 \\ -x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 4x_2 + 1}{5x_1 + 7x_2 + 1}.$$

Döntsük el, hogyan változna meg a 32. maximum- ill. minimum-feladat megoldása, ha a feltételrendszerben szerepelne az  $x_1 \leq 7$  feltétel is!

$$33. \quad E: \begin{cases} -2x_1 + x_2 \leq 6 \\ x_1 + x_2 \geq 5 \\ x_1 - x_2 \leq 3 \\ x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 8x_2 - 5}{x_1 + 5x_2 + 3}$$

$$34. \quad E: \begin{cases} 7x_1 + 6x_2 \leq 42 \\ 5x_1 - 3x_2 \geq -15 \\ 2x_1 + x_2 \geq 2 \\ 3x_1 - 4x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{9x_1 + 4x_2 - 1}{5x_1 + 2x_2 + 1};$$

$$35. \quad E: \begin{cases} 2x_1 + x_2 \geq 2 \\ -5x_1 + 3x_2 \leq 15 \\ 3x_1 - 4x_2 \leq 12 \\ 7x_1 + 6x_2 \leq 42 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{6x_1 + 5x_2 + 6}{5x_1 + 4x_2 + 4};$$

$$36. \quad E: \begin{cases} x_1 + 3x_2 \geq 3 \\ -4x_1 + 3x_2 \leq 12 \\ 3x_1 - 7x_2 \leq 21 \\ 7x_1 + 11x_2 \leq 77 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 + 1}{8x_1 + 5x_2 + 1}.$$

$$37. \quad E: \begin{cases} 6x_1 + 7x_2 \leq 42 \\ 3x_1 - 5x_2 \leq 15 \\ -4x_1 + 3x_2 \leq 12 \\ x_1 + 2x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{4x_1 + 9x_2 - 1}{2x_1 + 5x_2 + 1}.$$

$$38. \quad E: \begin{cases} 7x_1 + 6x_2 \leq 42 \\ -5x_1 + 3x_2 \leq 15 \\ 3x_1 - 4x_2 \leq 12 \\ 2x_1 + x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{5x_1 + 2x_2 + 1}{9x_1 + 4x_2 - 1}.$$

$$39. \quad E: \begin{cases} 3x_1 - 4x_2 \leq 12 \\ 11x_1 + 7x_2 \leq 77 \\ 3x_1 + x_2 \geq 3 \\ -7x_1 + 3x_2 \leq 21 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 3x_2 + 1}{5x_1 + 8x_2 + 1}.$$

$$40. \quad E: \begin{cases} x_1 - 4x_2 \leq 4 \\ -3x_1 + 4x_2 \leq 12 \\ 3x_1 + 4x_2 \geq 12 \\ x_1 \geq 2 \\ x_2 \leq 6 \\ x_1 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 13x_2 - 1}{x_1 + 5x_2 + 1}.$$

$$41. \quad E: \begin{cases} -7x_1 + 3x_2 \leq 21 \\ 3x_1 - 4x_2 \leq 12 \\ 3x_1 + x_2 \geq 3 \\ 11x_1 + 7x_2 \leq 77 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{5x_1 + 8x_2 + 1}{2x_1 + 3x_2 + 1}$$

$$42. \quad E: \begin{cases} -4x_1 + x_2 \leq 4 \\ 4x_1 - 3x_2 \leq 12 \\ 4x_1 + 3x_2 \geq 12 \\ x_2 \geq 2 \\ x_1 \leq 6 \\ x_2 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{5x_1 + x_2 + 1}{13x_1 + 3x_2 - 1}$$

$$43. \quad E: \begin{cases} -3x_1 + 5x_2 \leq 15 \\ 6x_1 + 5x_2 \leq 60 \\ x_1 + x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 + 1}{7x_1 + 5x_2 + 1} \rightarrow \max.$$

$$44. \quad E: \begin{cases} x_1 - 2x_2 \leq 8 \\ x_1 + x_2 \geq 6 \\ x_1 \geq 3 \\ -x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{5x_1 + 7x_2 + 1}{x_1 + x_2 + 1}$$

$$45. \quad E: \begin{cases} -x_1 + x_2 \geq -3 \\ x_1 + x_2 \geq 5 \\ -2x_1 + x_2 \leq 6 \\ x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{5x_1 + 8x_2 - 2}{2x_1 + 5x_2 + 1}$$

$$46. \quad E: \begin{cases} 2x_1 - x_2 \geq -6 \\ x_1 + x_2 \geq 5 \\ x_1 - x_2 \leq 3 \\ x_2 \geq 2 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{7x_1 + 5x_2 + 1}{4x_1 + 3x_2 + 1}$$

$$47. \quad E: \begin{cases} x_1 - x_2 \leq 6 \\ 3x_1 + 2x_2 \leq 24 \\ -x_1 + 3x_2 \leq 6 \\ x_1 + x_2 \geq 3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{4x_1 + 3x_2 - 6}{5x_1 + 2x_2 + 3}$$

$$48. \quad E: \begin{cases} -x_1 + x_2 \leq 3 \\ 3x_1 - x_2 \leq 3 \\ 2x_1 + 3x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 3x_2 + 10}{x_1 + x_2 + 4}$$

$$49. \quad E: \begin{cases} 2x_1 - 3x_2 \leq 6 \\ -2x_1 + x_2 \leq 2 \\ x_1 + 6x_2 \geq 6 \\ x_2 \leq 8 \\ x_1 \leq 7 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-5x_1 + 2x_2 - 31}{3x_1 + x_2 + 1}$$

$$50. \quad E: \begin{cases} x_1 + x_2 \geq 3 \\ 6x_1 + 5x_2 \leq 60 \\ -3x_1 + 5x_2 \leq 15 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 - 11}{8x_1 + x_2 + 1}$$

$$51. \quad E: \begin{cases} 5x_1 + 6x_2 \leq 60 \\ x_2 \leq 5 \\ 2x_1 + x_2 \geq 2 \\ 5x_1 - 3x_2 \leq 15 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 3x_2 - 1}{x_1 + x_2 + 2}$$

$$52. \quad E: \begin{cases} 4x_1 - 10x_2 \leq 20 \\ 4x_1 + 3x_2 \geq 12 \\ -8x_1 + 3x_2 \leq 24 \\ x_1 + x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 - 7}{4x_1 + x_2 + 4}$$

$$53. \quad E: \begin{cases} 5x_1 + 6x_2 \leq 60 \\ x_1 + x_2 \geq 2 \\ 5x_1 - 3x_2 \leq 15 \\ x_1 \leq 10 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{10x_1 + 14x_2 + 2}{2x_1 + 3x_2 + 1}$$

$$54. \quad E: \begin{cases} -3x_1 + 5x_2 \leq 15 \\ 6x_1 + 5x_2 \leq 60 \\ x_1 + x_2 \geq 2 \\ x_2 \leq 10 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{7x_1 + 5x_2 + 1}{3x_1 + 2x_2 + 1}$$

$$55. \quad E: \begin{cases} 10x_1 + 11x_2 \leq 110 \\ x_1 + x_2 \geq 5 \\ x_1 + 4x_2 \geq 8 \\ 3x_1 + x_2 \geq 9 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-2x_1 - 3x_2 + 1}{5x_1 + 9x_2 + 2}$$

$$56. \quad E: \begin{cases} x_1 - 3x_2 \leq 3 \\ -4x_1 + x_2 \leq 4 \\ x_1 + 2x_2 \geq 8 \\ 3x_1 + 2x_2 \geq 12 \\ x_1 \leq 12 \\ x_2 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + 2x_2 + 7}{x_1 + x_2 + 6}$$

$$57. \quad E: \begin{cases} -3x_1 + x_2 \leq 3 \\ 2x_1 + x_2 \leq 8 \\ 2x_1 + 3x_2 \geq 12 \\ x_1 \leq 8 \\ x_2 \leq 12 \\ x_1 - 4x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + x_2 + 6}{2x_1 + x_2 + 7}.$$

$$58. \quad E: \begin{cases} 3x_1 + 2x_2 \geq 12 \\ -4x_1 + x_2 \leq 4 \\ x_1 + 2x_2 \geq 8 \\ x_1 - 3x_2 \leq 3 \\ x_1 \leq 12 \\ x_2 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-x_1 - 2x_2 - 7}{x_1 + x_2 + 6}.$$

$$59. \quad E: \begin{cases} x_1 - 3x_2 \leq 3 \\ -4x_1 + x_2 \leq 4 \\ x_1 + 2x_2 \geq 8 \\ 3x_1 + 2x_2 \geq 12 \\ x_1 \leq 12 \\ x_2 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + 2x_2 + 7}{x_1 + x_2 + 6}.$$

$$60. \quad E: \begin{cases} -3x_1 + x_2 \leq 3 \\ x_1 - 4x_2 \leq 4 \\ 2x_1 + x_2 \geq 8 \\ 2x_1 + 3x_2 \geq 12 \\ x_1 \leq 8 \\ x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + x_2 + 6}{2x_1 + x_2 + 7}.$$

$$61. \quad E: \begin{cases} x_1 - 2x_2 \leq 8 \\ x_1 + x_2 \geq 6 \\ x_1 \geq 3 \\ -x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 3x_2 - 11}{x_1 + 8x_2 + 1}.$$

$$62. \quad E: \begin{cases} x_1 - 2x_2 \leq 8 \\ x_1 + x_2 \geq 6 \\ x_1 \geq 3 \\ -x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-5x_1 + 2x_2 - 9}{7x_1 + 3x_2 + 1}.$$

$$63. \quad E: \begin{cases} x_1 + 2x_2 \geq 2 \\ -4x_1 + 3x_2 \leq 12 \\ 3x_1 - 5x_2 \leq 15 \\ 6x_1 + 7x_2 \geq 42 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{4x_1 + 5x_2 + 4}{5x_1 + 6x_2 + 6}.$$

$$64. \quad E: \begin{cases} -7x_1 + 3x_2 \leq 21 \\ 4x_1 + 3x_2 \geq 12 \\ 2x_1 - 5x_2 \leq 10 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 - 3x_2 - 24}{x_1 + 2x_2 + 2}.$$



$$65. \quad E: \begin{cases} 3x_1 - 7x_2 \leq 21 \\ -5x_1 + 2x_2 \leq 10 \\ x_2 \leq 10 \\ 3x_1 + 4x_2 \geq 12 \\ x_1 \leq 7 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 5x_2 + 1}{x_1 + 3x_2 + 1}.$$

$$66. \quad E: \begin{cases} -3x_1 + 4x_2 \leq 0 \\ x_1 + x_2 \geq 2 \\ 3x_1 + 4x_2 \leq 24 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-x_1 + x_2 - 7}{x_1 + 2x_2 + 1}.$$

$$67. \quad E: \begin{cases} 2x_1 - 5x_2 \leq 10 \\ -7x_1 + 3x_2 \leq 21 \\ x_2 \leq 7 \\ 4x_1 + x_2 \geq 12 \\ x_1 \leq 10 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + x_2 + 1}{5x_1 + 2x_2 + 1}.$$

$$68. \quad E: \begin{cases} -x_1 + 3x_2 \leq 3 \\ x_1 - x_2 \leq 3 \\ 3x_1 + 2x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + x_2 + 4}{2x_1 + 3x_2 + 10}.$$

$$69. \quad E: \begin{cases} 3x_1 - x_2 \leq 6 \\ -x_1 + x_2 \leq 6 \\ 2x_1 + 3x_2 \leq 24 \\ x_1 + x_2 \geq 3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 4x_2 - 6}{2x_1 + 5x_2 + 3}.$$

$$70. \quad E: \begin{cases} x_1 + x_2 \leq 12 \\ 3x_1 + 4x_2 \geq 12 \\ -5x_1 + 2x_2 \leq 10 \\ 3x_1 - 8x_2 \leq 24 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 3x_2 - 7}{x_1 + 4x_2 + 4}.$$

$$71. \quad E: \begin{cases} x_1 + x_2 \leq 5 \\ 4x_1 + x_2 \geq 8 \\ x_1 + 3x_2 \geq 9 \\ 11x_1 + 10x_2 \leq 110 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 + 2x_2 - 1}{9x_1 + 5x_2 + 2}.$$

$$72. \quad E: \begin{cases} 2x_1 + 3x_2 \geq 6 \\ 10x_1 + 11x_2 \leq 110 \\ x_2 \leq 8 \\ x_1 \leq 9 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 + 7x_2 + 2}{3x_1 + 10x_2 + 2}.$$

$$73. \quad E: \begin{cases} x_1 + 2x_2 \leq 9 \\ 3x_1 + 2x_2 \geq 6 \\ x_1 - x_2 \leq 3 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-2x_1 - x_2 + 1}{3x_1 + x_2 + 2}$$

$$74. \quad E: \begin{cases} x_1 - x_2 \geq -3 \\ 2x_1 + x_2 \leq 9 \\ 2x_1 + 3x_2 \geq 6 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-x_1 - 2x_2 + 1}{x_1 + 3x_2 + 2}$$

$$75. \quad E: \begin{cases} -2x_1 + x_2 \leq 2 \\ 3x_1 + 2x_2 \geq 6 \\ 2x_1 - x_2 \leq 8 \\ x_1 + 5x_2 \geq 5 \\ x_2 \leq 8 \\ x_1 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + 5x_2 - 5}{5x_1 + 12x_2 + 1}$$

$$76. \quad E: \begin{cases} 5x_1 + 8x_2 \leq 40 \\ 3x_1 + 2x_2 \leq 6 \\ x_1 - 2x_2 \leq 4 \\ -x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-5x_1 + 3x_2 + 10}{3x_1 + 2x_2 + 13}$$

$$77. \quad E: \begin{cases} -2x_1 + x_2 \leq 2 \\ 2x_1 - x_2 \leq 8 \\ 3x_1 + 2x_2 \geq 6 \\ x_1 + 5x_2 \geq 5 \\ x_2 \leq 9 \\ x_1 \leq 7 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{2x_1 - x_2 - 8}{5x_1 + 2x_2 + 7}$$

$$78. \quad E: \begin{cases} x_1 - 2x_2 \leq 4 \\ x_1 + 4x_2 \geq 8 \\ 3x_1 - x_2 \geq -6 \\ 5x_1 + x_2 \geq 5 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 - 2x_2 - 4}{5x_1 + 11x_2 + 1}$$

$$79. \quad E: \begin{cases} -x_1 + x_2 \leq 3 \\ x_1 + 10x_2 \geq 10 \\ 4x_1 + x_2 \leq 8 \\ x_1 + x_2 \leq 7 \\ 3x_1 - 4x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 + 10x_2 - 10}{2x_1 + 7x_2 + 6}$$

$$80. \quad E: \begin{cases} x_1 + 5x_2 \geq 5 \\ x_1 - 3x_2 \leq 6 \\ 4x_1 + x_2 \geq 8 \\ -2x_1 + x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-2x_1 + x_2 - 4}{11x_1 + 5x_2 + 1}$$

$$81. \quad E: \begin{cases} x_1 - 2x_2 \leq 2 \\ -x_1 + 2x_2 \leq 8 \\ 2x_1 + 3x_2 \geq 6 \\ x_1 + 5x_2 \geq 5 \\ x_2 \leq 8 \\ x_1 \leq 8 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{5x_1 + x_2 - 5}{12x_1 + 5x_2 + 1}$$

$$82. \quad E: \begin{cases} 8x_1 + 5x_2 \leq 40 \\ -2x_1 + x_2 \leq 4 \\ 2x_1 + 3x_2 \geq 6 \\ x_1 - x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{3x_1 - 5x_2 + 10}{2x_1 + 3x_2 + 13}$$

$$83. \quad E: \begin{cases} x_1 - 2x_2 \leq 2 \\ 2x_1 + 3x_2 \geq 6 \\ -x_1 + 2x_2 \leq 8 \\ 5x_1 + x_2 \geq 5 \\ x_2 \leq 7 \\ x_1 \leq 9 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-x_1 + 2x_2 - 8}{2x_1 + 5x_2 + 7}$$

$$84. \quad E: \begin{cases} -2x_1 + x_2 \leq 4 \\ -x_1 + 3x_2 \geq -6 \\ 4x_1 + x_2 \geq 8 \\ x_1 + 5x_2 \geq 5 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{-2x_1 + x_2 - 4}{11x_1 + 5x_2 + 1}$$

$$85. \quad E: \begin{cases} x_1 - x_2 \leq 3 \\ x_1 + 4x_2 \geq 8 \\ 10x_1 + x_2 \geq 10 \\ x_1 + x_2 \leq 7 \\ -4x_1 + 3x_2 \leq 12 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{10x_1 + x_2 - 10}{7x_1 + 2x_2 + 6}$$

$$86. \quad E: \begin{cases} 5x_1 + x_2 \geq 5 \\ x_1 + 4x_2 \geq 8 \\ -3x_1 + x_2 \leq 6 \\ x_1 - 2x_2 \leq 4 \\ \mathbf{x} \geq 0 \end{cases}$$

$$z(\mathbf{x}) = \frac{x_1 - 2x_2 - 4}{5x_1 + 11x_2 + 1}$$